The 3rd Japan-China Geometry Conference

Date : Sept. 1st (Fri) $\sim$ 7th (Th)
Venue : Tohoku Forum for Creativity, Tohoku University

Title and Abstract

Bing-Long Chen (Sun Yat-sen University)

Title: On negatively curved manifolds

Abstract: The purpose of this talk is to report our recent works related to the geometry of negatively curved manifolds. These results are related to a conjecture of Hopf, which states that for a compact even $2n$-dimensional Riemannian manifold X with negative sectional curvature, its Euler charateristic number satisfies $(-1)^n \chi(X) > 0$.

Qing Ding (Fudan University)

Title: Nonpositively curved manifolds and the Martin boundary

Abstract: The Martin representation formula for harmonic functions on a complete simply-connected Riemannian manifold with negatively pinched curvature has been established by Anderson and Schoen. In this talk, we try to generalize it geometrically to the case of non-positive and non-pinched negative curvature.

Akito Futaki (University of Tokyo)

Title: Structure of automorphism group of conformally Einstein-Maxwell Kähler manifolds

Abstract: We prove an extension of Lichnerowicz-Matsushima theorem to conformally Einstein-Maxwell Kähler manifolds. More generally, we prove an extension of Calabi's theorem for extremal Kähler manifolds. The proof uses Hessian formula in the Donaldson-Fujiki type setup. This is a joint work with Hajime Ono.

Shouhei Honda (Tohoku University)

Title: New stability results for sequences of metric measure spaces with uniform Ricci bounds from below

Abstract: In this talk we introduce several stability results with respect to the measured Gromov-Hausdorff convergence under assuming uniform lower bounds on Ricci curvature. In particular we prove stability of Sobolev/BV functions, Hessians, and generalized Bakry-Emery conditions. Applications include a suspension theorem for positively Ricci curved spaces, which is new even for smooth manifolds. More recent works are also discussed if there is sufficient time. This is a joint work with Luigi Ambrosio.

Bobo Hua (Fudan University)

Title: Combinatorial curvature for planar graphs

Abstract: The combinatorial curvature of a planar graph is defined as the generalized Gaussian curvature of its polygonal surface with a piecewise

flat metric. We will show that the total curvature of a planar graph, whose faces are isometric to regular polygons in the Euclidean plane, with nonnegative combinatorial curvature is an integral multiple of $1/6 * \pi$. This is a joint work with Yanhui Su.

Yu Kawakami (Kanazawa University)

Title: Global properties for the Gauss images of various classes of surfaces

Abstract: We explain our systematic study of global properties for the images of the Gauss maps of several classes of surfaces. In particular, we give geometric interpretations of the maximal number of omitted values of the Gauss maps of complete minimal surfaces in Euclidean space and the ratio of canonical forms of weakly complete flat fronts in hyperbolic 3-space.

Anmin Li (Sichuan University)

Title: Virtual Neighborhood Technique for Holomorphic Curve Moduli Spaces

Abstract: Ruan and Tian established the theory of Gromov-Witten invariants for semi-positive symplectic manifold, about 90's. To remove semi-positive condition, the technology went on a significant change. There had been several different approaches for general symplectic manifolds, such as Fukaya-Ono, Li-Tian, Liu-Tian, Ruan, Siebert and etc. Recently, there is a great deal of interest to re-visit the latter approach with the purpose to clean up some of issues. The main complication is that the moduli space has various lower strata. How to deal with these lower strata is one of main issues discussed recently. Our main idea is that the Gromov-Witten invariants can be defined as an integral over top strata of virtual neighborhood. Therefore, all the complication near lower strata of the moduli space can be avoided entirely. The invariants defined in this way satisfy all the Gromov-Witten axioms of Kontsevich and Manin. In this paper we use the approach of Ruan and Li-Ruan with some modifications.

Remark: In the original approach of Ruan and Li-Ruan the authors first deformed ω such that $[\omega]$ is the Chern class of a complex line bundle L over M. Then they considered the line bundle $u^*L \otimes \omega_\Sigma$. In our present approach we do not deform M. The arguments of our approach are more simple and can be applied to the case when M is an orbifold (joint work with Li Sheng).

Jiayu Li (The University of Science and Technology of China)

Title: Canonical metrics and The Hermitian-Yang-Mills flow on reflexive sheaves

Abstract: In this talk, we will introduce our recent work on the existence of canonical metrics, Bogomolov type inequalities and the limiting behavior of the Hermitian-Yang-Mills flow on reflexive sheaves. These are joint work with Xi Zhang and Chuanjing Zhang.

Hui Ma (Tsinghua University)

Title: Uniqueness of closed self-similar solutions to σ_k^α-curvature flow

Abstract: By adapting the test functions introduced by Choi-Daskaspoulos and Brendle-Choi-Daskaspoulos and exploring properties of the k-th elementary symmetric functions σ_k intensively, we show that for any fixed k with $1 \le k \le n-1$, any strictly convex closed hypersurface in $\mathbb{R}^{n+1}$ satisfying $\sigma_k^\alpha = \langle X, \nu \rangle$, with $\alpha \ge \frac{1}{k}$, must be a round sphere. In fact, we prove a

uniqueness result for any strictly convex closed hypersurface in $\mathbb{R}^{n+1}$ satisfying $F + C = \langle X, \nu \rangle$, where F is a positive homogeneous smooth symmetric function of the principal curvatures and C is constant. The talk is based on the recent joint work with Shanze Gao and Haizhong Li.

Toshiki Mabuchi (Osaka University)

Title: The Chow norm and the existence problem of extremal metrics

Abstract: The Yau-Tian-Donaldson conjecture for anti-canonical polarization was recently solved affirmatively by Chen-Donaldson-Sun and Tian. However, this conjecture is still open for general polarizations or more generally in extremal Kähler cases. In this talk, extremal Kähler versions of the conjecture will be discussed. It will be shown that the problem is closely related to the geometry of moduli spaces of test configurations for polarized algebraic manifolds. Another important tool in our approach is the Chow norm. This is closely related to Ding's functional, and plays a crucial role in our differential geometric study of stability. By discussing the Chow norm from various points of view, we shall make a systematic study of the conjecture.

Shin Nayatani (Nagoya University)

Title: Metrics on a closed surface of genus two which maximize the first eigenvalue of the Laplacian

Abstract: For any Riemannian metric of area one on a closed orientable surface of genus two, the first eigenvalue of the Laplacian is bounded from above by 16π. Jakobson-Levitin-Nadirashvili-Nigam-Polterovich conjectured that a certain singular metric on the Bolza surface should attain the maximum value. In this talk, we shall discuss a proof of this conjecture. This is a joint work with Toshihiro Shoda of Saga University.

Yuji Sano (Fukuoka University)

Title: A moment map for relative balanced metrics

Abstract: Donaldson gave a general framework to study some specific Fubini-Study metrics on a polarized manifold, which are called balanced metrics. As an application, he gave the quantization of the constant scalar curvature metrics by balanced metrics. In this talk, I explain the extension of Donaldson's framework to some relative balanced metrics and its application to the extremal Kähler metrics. This is a joint work with Carl Tipler.

Guangxiang Su (Chern Institute of Mathematics)

Title: Positive scalar curvature and connected sums

Abstract: Let N be a closed enlargeable manifold in the sense of Gromov-Lawson and M a closed spin manifold of equal dimension. A famous theorem of Gromov-Lawson states that the connected sum $M\#N$ admits no metric of positive scalar curvature. We present a potential generalization of this result to the case where M is nonspin. We use index theory for Dirac operators to prove our result. This is a joint work with Professor Weiping Zhang.

Ryokichi Tanaka (Tohoku University)

Title: Harmonic measures for negatively curved manifolds

Abstract: We establish a dimension formula for the harmonic measure on the ideal boundary of a Riemannian manifold when it is a regular covering of a

finite volume manifold and hyperbolic in the sense of Gromov. In particular we show that the harmonic measure has a positive dimension.

Gang Tian (Peking University, Princeton University)

Title: Conic Kahler-Einstein metrics

Zhenxiao Xie (China University of Mining and Technology)

Title: Three Dimensional Conformally Flat Lorentzian Hypersurfaces in Lorentzian Space Forms

Abstract: A three dimensional Lorentzian hypersurface $x : M_1^3 \to \mathbb{N}_1^4(c)$ is called conformally flat if its induced metric is conformal to the flat Lorentzian metric. This property is preserved under the conformal transformation of $\mathbb{N}_1^4(c)$. It is an analogy to conformally flat hypersurfaces in Riemannian space forms (studied firstly by Cartan), but more complicated since its shape operator may fail to be diagonalizable. In this talk, we will discuss some recent works about these hypersurfaces. If the shape operator has three distinct eigenvalues (real or complex), by constructing a scalar conformal invariant, we obtain some systems of partial differential equations, of which solutions determine these hypersurfaces. If the shape operator has repeated eigenvalues, we get a complete classification. This is a joint work with Changping Wang and Xiaozhen Wang.

Naoto Yotsutani (Nagoya University)

Title: On relative GIT stability of toric Fano varieties

Abstract: In this talk, we discuss " relative GIT stability " for toric polarizations in order to deal with extremal metrics. A part of this talk is based on the joint work with Bin Zhou.

Weiping Zhang (Chern Institute of Mathematics)

Title: The Chern conjecture for affine manifolds

Abstract: We present the joint work with Huitao Feng on the proof of the Chern conjecture which states that the Euler characteristic of a closed affine manifold equals zero.

Xi Zhang (The University of Science and Technology of China)

Title: Complex Monge-Ampère equation and its applications

Abstract: The complex Monge-Ampère equation has been the subject of intensive studies in the past forty years, since its significant applications in complex analysis and complex geometry. In this talk, I will introduce our recent work on regularity estimates of the complex Monge-Ampère equation and its applications in Kähler geometry and Sasakian geometry.

Yongsheng Zhang (Northeast Normal University)

Title: Minimizing cones associated with isoparametric foliations

Abstract: Associated with isoparametric foliations of unit spheres, there are two classes of minimal submanifolds — minimal isoparametric hypersurfaces and focal submanifolds. By virtue of their rich structures, we find new series of minimizing cones. They are cones over focal submanifolds and cones over suitable products among these two classes. Except in low dimensions, we show that all such cones are minimizing. The talk is based on a joint work with Professor Zizhou Tang.

Zhenlei Zhang (Capital Normal University)

Title: Analytic Minimal Model Program through Continuity Method

Abstract: One continuity method of complex Monge-Ampère equations was introduced by La Nave and Tian to run the Analytic Minimal Model Program. In this talk, I will review the recent progress in this program. The results in the talk are based on joint work with La Nave, Tian, Y. Zhang.

Xiangyu Zhou (Chinese Academy of Sciences)

Title: Some results on multiplier ideal sheaves and optimal L^2 extensions

Abstract: In this talk, we'll report our recent work about multiplier ideal sheaves and optimal L^2 extensions.